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## PROBLEMS AND SOLUTIONS.

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## PROBLEMS FOR SOLUTION.

**2806. Proposed by WARREN WEAVER, Throop College of Technology.**

The Maxwell law for the distribution of velocities among the molecules of a gas shows that, considering a velocity of given magnitude, it is equally probable that it have any direction. Find an expression for the probability that the velocity of a molecule  $m_2$  (whose velocity relative to the mass motion of the gas is  $k'$ ) when measured relative to the velocity  $k$  of a molecule  $m_1$  ( $k$  less than  $k'$ ) shall make with  $k$  an angle falling within the limits  $\theta$  and  $\theta + d\theta$ .

**2807. Proposed by S. A. COREY, Des Moines, Iowa.**

Establish the identity

$$\begin{vmatrix} -x & bcy & -acu & -abv \\ y & x & -av & au \\ u & bv & x & by \\ -v & cu & cy & -x \end{vmatrix}^2 + \begin{vmatrix} x & bcy & -x & -abv \\ -y & x & y & au \\ u & bv & u & by \\ -v & cu & -v & -x \end{vmatrix}^2 + \begin{vmatrix} ab & x & bcy & -acu & -x \\ -y & x & -av & y & x \\ u & bv & x & u & u \\ -v & cu & cy & -v & v \end{vmatrix}^2 = \begin{vmatrix} x & bcy & -acu & -abv \\ -y & x & -av & au \\ u & bv & x & by \\ -v & cu & cy & -x \end{vmatrix}^2$$

**2808. Proposed by A. A. BENNETT, University of Texas.**Let  $f(x)$  denote an arbitrary frequency distribution function.

(1) Show that the condition that frequencies when compounded follow the law for simple frequencies is that the iteration of  $f(r-t)$  in the sense of integral equations leaves this kernel unaltered, i.e.,

$$(i) f(r-t) = \int_{-\infty}^{+\infty} f(r-s)f(s-t)ds \text{ and, hence, this kernel is singular.}$$

Cf. David F. Barrow, *Annals of Mathematics*, second series, vol. 19 (1917), p. 97.

(2) Show that  $f(a, x)$  satisfies (i) for every real value of  $a$ , when

$$f(a, x) = \begin{cases} \frac{1}{\pi x} \sin ax, & \text{for } -c < x < c, \text{ } c \text{ being arbitrary, real and positive but fixed. In par-} \\ & \text{ticular } c \text{ may be infinite.} \\ 0, & \begin{cases} \text{for } x < -c. \\ \text{for } x > c. \end{cases} \end{cases}$$

(3) Show that for a given  $c$ , and  $0 < a < b$ ,  $f(b, x) - f(a, x)$  and  $f(a, x)$  are orthogonal.

(4) Show that the Gaussian normal probability function satisfies (i). Consider its expansion in terms of the orthogonal functions of (3).

(5) Show that for  $c$  finite,  $f(a, x)$  cannot be defined by means of a Fourier series, save for discrete values of  $a$ , since for  $a$  varying within certain restricted segments, the Fourier coefficients of  $f(a, x)$  remain unaltered. Show that when a Fourier representation is possible, the series reduces to a trigonometric polynomial.

**2809. Proposed by the late L. G. WELD.**Find the  $n$ th term of the series defined by the relation,  $u_{i+2} = u_i + u_{i+1}$ , in which  $u_1 = u_2 = 1$ .

**2810. Proposed by H. S. UHLER, Yale University.**

In the expansion of the following determinant or eliminant, find the total number of terms, and the number of terms having the coefficients  $+1, -1, +2, -2, +3, -3, +4, -4, +5, -5, +6, -8, +10$ , respectively.

$$\begin{vmatrix} a & b & c & d & e & 0 & 0 & 0 \\ 0 & a & b & c & d & e & 0 & 0 \\ 0 & 0 & a & b & c & d & e & 0 \\ 0 & 0 & 0 & a & b & c & d & e \\ A & B & C & D & E & 0 & 0 & 0 \\ 0 & A & B & C & D & E & 0 & 0 \\ 0 & 0 & A & B & C & D & E & 0 \\ 0 & 0 & 0 & A & B & C & D & E \end{vmatrix}$$

**2811. Proposed by J. L. RILEY, Stephenville, Texas.**

Given the cube roots of 60, 61, 63, and 64, to find the cube root of 62 by the method of differences.

**2812. Proposed by C. N. SCHMALL, New York City.**

If  $F(x, y, z)$  be a homogeneous function of  $x, y, z$ , which becomes  $\phi(u, v, w)$  by the elimination of  $x, y, z$ , by means of the equations  $\partial F/\partial x = u, \partial F/\partial y = v, \partial F/\partial z = w$ ; show that

$$\frac{\partial F}{\partial u} \bigg/ x = \frac{\partial F}{\partial v} \bigg/ y = \frac{\partial F}{\partial w} \bigg/ z.$$

**2813. Proposed by PAUL CAPRON, U. S. Naval Academy.**

An ellipse having the major-axis  $2a$  and the eccentricity  $\epsilon$ , is revolved first about its major axis, forming a prolate spheroid, then about its minor axis forming an oblate spheroid. Show that the surfaces of these spheroids are, respectively,

$$2\pi a^2(1/\epsilon \sqrt{1 - \epsilon^2} \sin^{-1} \epsilon + 1)$$

and

$$2\pi a^2 \left[ 2 + 1/\epsilon(1 - \epsilon^2) \log \left( \frac{1 + \epsilon}{1 - \epsilon} \right) \right].$$

**SOLUTIONS OF PROBLEMS.****339 (Calculus) [June, 1913; May, 1919]. Proposed by T. H. GRONWALL, Washington, D. C.**

To show that for any real value of  $x$

$$\left| \frac{d^n}{dx^n} \left( \frac{\sin x}{x} \right) \right| \leq \frac{1}{n+1}, \quad \text{and} \quad \left| \frac{d^n}{dx^n} \left( \frac{1 - \cos x}{x} \right) \right| \leq \frac{1}{n+1}.$$

**I. SOLUTION BY OTTO DUNKEL, Washington University.**

The function  $y = \sin x/x, x \neq 0; y = 1, x = 0$ , is single-valued and continuous for all values of  $x$  and it can be expressed as a power series

$$y = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots$$

The power series shows that the function possesses all of its derivatives at  $x = 0$ , and it is easily seen from the power series or from the original form of the function that the derivatives exist for all other values of  $x$ . Hence, we may write the sequence of equations for finding the derivatives

$$xy = \sin x, \quad y + xy' = \sin \left( \frac{\pi}{2} + x \right),$$

$$2y' + xy'' = \sin(\pi + x), \quad \dots, \quad (n+1)y^{(n)} + xy^{(n+1)} = \sin \left( \frac{n+1}{2} \pi + x \right), \quad \dots$$